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Solution		

Prob #	1	2	3	4	Total
Points	25	25	25	25	

Time: 80 Minutes

Solution

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$$F(\mathbf{x}) = F(\mathbf{x}^*) + \nabla F(\mathbf{x})^T \Big|_{\mathbf{x} = \mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) \\ + \frac{1}{2} (\mathbf{x} - \mathbf{x}^*)^T \nabla^2 F(\mathbf{x}) \Big|_{\mathbf{x} = \mathbf{x}^*} (\mathbf{x} - \mathbf{x}^*) + \dots$$

$$\frac{\mathbf{p}^T \nabla F(\mathbf{x})}{\|\mathbf{p}\|} \quad \frac{\mathbf{p}^T \nabla^2 F(\mathbf{x}) \mathbf{p}}{\|\mathbf{p}\|^2} \quad \alpha_k = -\frac{\mathbf{g}_k^T \mathbf{p}_k}{\mathbf{p}_k^T \mathbf{A} \mathbf{p}_k}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \mathbf{g}_k \quad \mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$L_i = \sum_{j \neq i} \max(0, y_j - y_i + \Delta)$$

$$S(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

$$H(p,q)=-\sum_x p(x)\log(q(x))$$

$$L_i = -\log\left(\frac{e^{y_i}}{\sum_j e^{y_j}}\right)$$

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1. Consider the following performance surface

$$F(X) = 2x_1^2 - 3x_1x_2 + 5x_1 - 4x_2$$

Given the initial point $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, take **two steps** of the **steepest descent algorithm**, minimizing along a line **at each step**.

You must show the resulting position after each step.

$$\nabla F(x)|_{X=X_0} = \begin{bmatrix} 4x_1 - 3x_2 + 5 \\ -3x_1 - 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$\nabla^2 F(x)|_{X=X_0} = \begin{bmatrix} 4 & -3 \\ -3 & 0 \end{bmatrix}$$

$$p_0 = -g_0 = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$$

$$\alpha_0 = - \frac{\begin{bmatrix} 10 & -10 \end{bmatrix} * \begin{bmatrix} -10 \\ 10 \end{bmatrix}}{\begin{bmatrix} -10 & 10 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -10 \\ 10 \end{bmatrix}} = - \frac{-200}{\begin{bmatrix} -10 & 10 \end{bmatrix} \begin{bmatrix} -70 \\ 30 \end{bmatrix}}$$

$$= - \frac{-200}{1000} = 0.2$$

$$X_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 0.2 \begin{bmatrix} -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$\nabla F(x)|_{X=X_0} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}$$

$$p_1 = -g_1 = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\alpha_1 = - \frac{\begin{bmatrix} -4 & -4 \end{bmatrix} * \begin{bmatrix} 4 \\ 4 \end{bmatrix}}{\begin{bmatrix} 4 & 4 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix}} = - \frac{-32}{\begin{bmatrix} 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -12 \end{bmatrix}} = - \frac{-32}{-32} = -1$$

$$X_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix} + (-1) \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$$

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Problem 1 Continued

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2. After executing the program below, what exactly is printed? Show the result below..

```
import torch
import torch.nn as nn

# Input: (batch, channels, height, width)
x = torch.randn(100, 3, 96, 140)

model = nn.Sequential(
    nn.Conv2d(in_channels=3, out_channels=20, kernel_size=(7,5), stride=(3,5),
padding=(2, 0), bias=False),
    nn.MaxPool2d(kernel_size=(2,2), stride=(2,2), padding=(0,0)),
    nn.Flatten(),
    nn.Linear(in_features=4480, out_features=60, bias=False))

with torch.no_grad():
    y0 = model[0](x)
    y1 = model[1](y0)
    y2 = model[2](y1)
    y3 = model[3](y2)

print("1:", tuple(model[0].weight.shape))
print("2:", tuple(y0.shape))
print("3:", tuple(y1.shape))
print("4:", tuple(y2.shape))
print("5:", tuple(model[3].weight.shape))
print("6:", tuple(y3.shape))
```

```
1: (20, 3, 7, 5)
2: (100, 20, 32, 28)
3: (100, 20, 16, 14)
4: (100, 4480)
5: (60, 4480)
6: (100, 60)
```

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Problem 2 Continued

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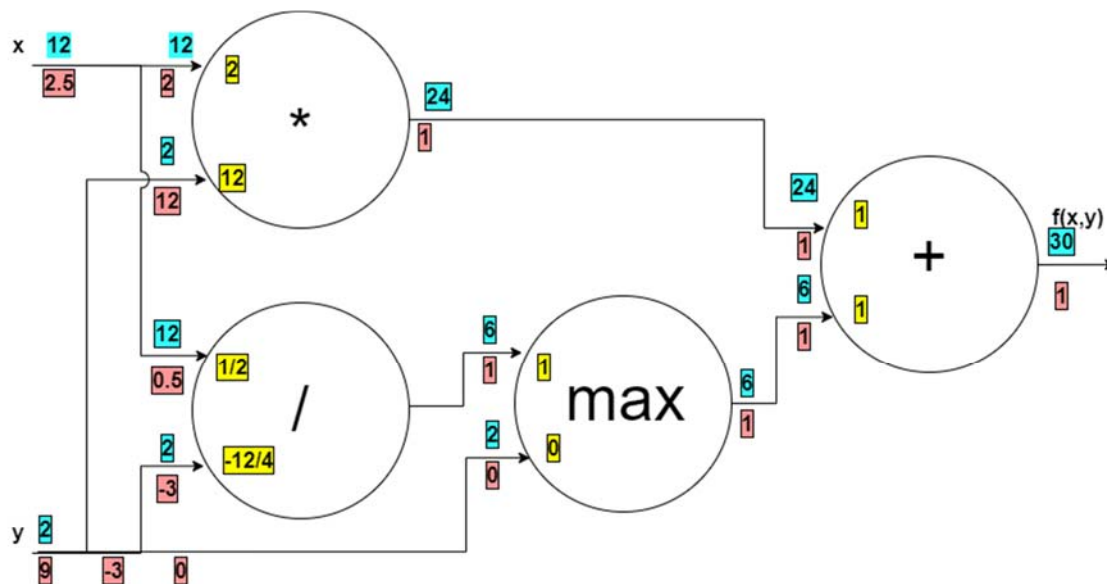
3. Consider the expression: $f(x, y) = (x * y) + \max(\frac{x}{y}, y)$

Given the inputs $x = 12$, $y = 2$

Draw the computational graph and calculate the $\frac{\delta f(x,y)}{\delta x}$ and $\frac{\delta f(x,y)}{\delta y}$

Show all the numerical values of the forward and backward pass.

You **MUST SHOW** all the numerical values as they flow in the forward and backward path in the computational graph.



Blue: Forward pass

Yellow: Local derivative

Red: Backward pass

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Problem 3 Continued

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4. Complete the code for the following function.

This function is **identical to the initialization step used in Assignment 01**.

```
import numpy as np
```

```
def initialize_multi_layer_nn(X_train,Y_train,layers):
```

```
    """ This function Initializes all the weights and returns weight  
    matrices in a list. You do not need to train the network.
```

```
    bias should be included in the weight matrix.
```

```
    X_train: numpy array of input shape=[input_dimension,num_train_samples]
```

```
    Y_train: numpy array of targets
```

```
    shape=[output_dimension,num_train_samples]
```

```
    layers: array of integers representing number of nodes in each layer
```

```
    return: This function should return a list of weight matrices.
```

```
    Each element of the list should be a numpy array corresponds to the  
    weight matrix of the corresponding layer.
```

```
    Initialize all the weights to zero """
```

```
        previous_dimension=X_train.shape[0]
```

```
        weight_matrices=[]
```

```
        for k in layers:
```

```
            temp_w=np.zeros((k,previous_dimension+1))
```

```
            weight_matrices.append(temp_w)
```

```
            previous_dimension=k
```

```
        return weight_matrices
```

```
#Another solution
```

```
    temp=np.append(X_train.shape[0],np.copy(layers))
```

```
    return [np.zeros((temp[k+1],temp[k]+1)) for k in range(len(temp)-1)]
```

